

Chapter 5 Inclined Plane Sliding

Summary

In this chapter experiments using an inclined plane sliding technique to determine angles of friction are described.

Experiments using wooden blocks of different geometries and metal weighted wooden sliders to study the effect of stress distribution on measured values of ϕ , show that the angle of sliding is dependant upon the geometry of the block (height: length ratio), and mass of the block, independantly. Experiments in which the geometry of the leading edge of the sliding block is varied show that blocks with pointed leading edges slide at lower angles than blocks with straight edges.

Experiments using rock sliders attached to blocks of different density materials and geometries are described. It is shown that for repeated sliding, the angle of sliding decreases with displacement for most rock types studied. The residual angle of sliding reached is shown to be dependant upon the presence of rock flour between the surfaces. The area of wear is shown to be directly related to stress distribution.

The initial steep drop in angle of sliding seen for most rock types in the early stages of testing, is accompanied by the generation of large amounts of rock flour, relative to that generated during later stages of testing. The residual angle of sliding for rock surfaces covered in rock flour is not dependant upon the grain size of the rock flour. Observation suggests that the dominant process involved in the decrease in angle of sliding is one of attrition involving the shearing and polishing of asperities.

The final section describes experiments on Darleydale sandstone sliders using 32 blocks of different geometries and densities. The relationships between geometry, mass and angle of sliding observed for wood are not seen for rock. Comparison of results with those from more conventional testing apparatus show good correlation for peak values and it is concluded that the inclined plane test is reliable and produces repeatable results.

Chapter 5. Inclined Plane Sliding

5.1 Introduction

In chapter 3 the design of a vibrating test rig is described. This design entails an inclined plane on which a block is placed in a stable position. Sliding is then induced by input of a horizontal vibratory force.

The idea of using an inclined plane to determine the angle of friction dates back to Leonardo da Vinci. The test is rarely used now, direct shear tests being preferred. This preference is due especially to limitations on the applied loads that may be used in inclined plane tests, Hencher (1976). Another reason is that the test which is used in many Physics courses as a standard teaching experiment is not generally regarded as being very repeatable, advice being given to tap the block in order to obtain less erratic results, Aharoni (1972). To the best knowledge of the author no experiments have been reported that investigate the cause of these erratic results.

The mathematical analysis of inclined plane sliding and its relation to Coulomb friction have been given in Chapter 3.

5.2 Apparatus

The apparatus used is illustrated diagrammatically in figure 5.1. The basal plane is raised by turning a screw attached to the plane by means of a ball and socket joint. The raising system is greased to prevent vibration and involves approximately three complete screw rotations for a single degree increase in inclination. Preparation of specimens and top block are described later.

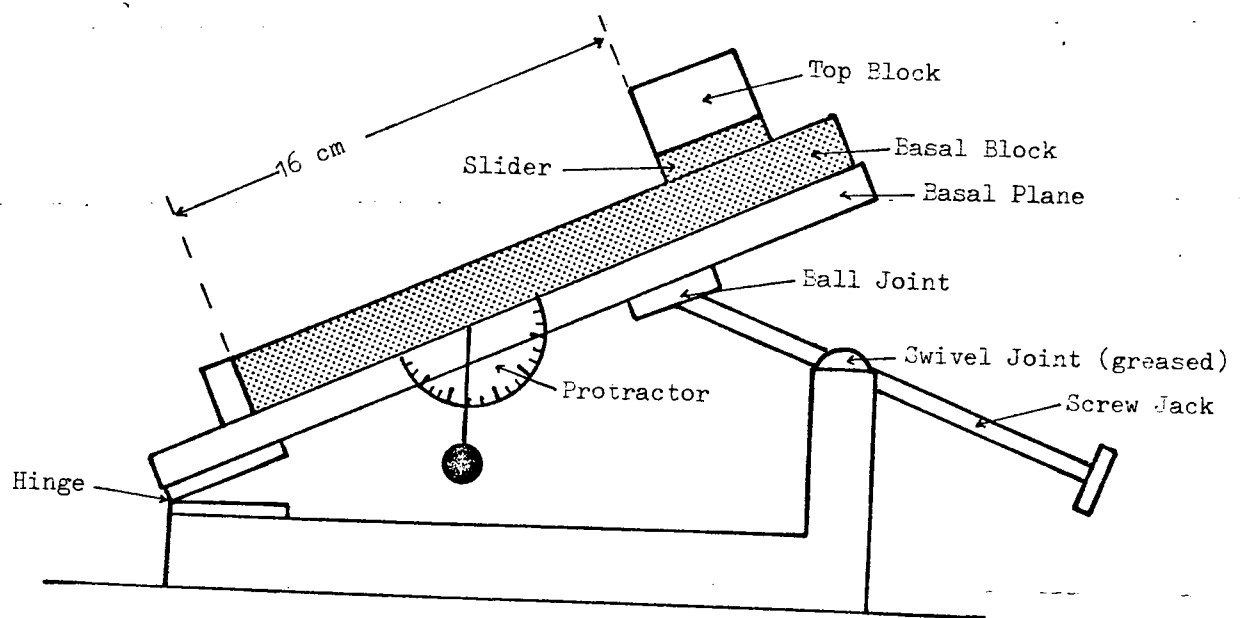
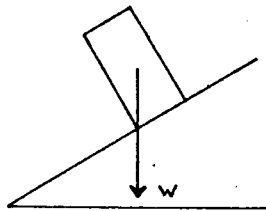


Figure 5.1 Apparatus used in inclined plane sliding experiments.

5.3 Stress Distribution along an Inclined Plane

Although components of load are simply controlled by the angle of inclination of the slope, stress distribution is more complex.

In an orthogonal block, resting on an inclined plane, it is clear that the total weight of the block acts through its leading edge at the moment of toppling, and the stress along that edge is theoretically infinitely large.



The distribution of stress along the base of the block will vary with the inclination of slope and the geometry and density distribution of the block. The stresses will be highest at the down-slope edge of the block and least at the rear edge.

A simple experiment to illustrate this fact may be made by placing a piece of paper under the rear half of a block on a plane and trying to remove it by pulling when the plane is horizontal and again when inclined. It is found that the paper may be more easily extracted from an inclined block.

A mathematical analysis of this stress distribution is given in Appendix 1.

The area of probable contact between the surfaces will be controlled by the position of the weight vector, assuming a random distribution of asperities. The majority of contacts will be made, therefore, towards the down-slope edge of the block and the shear strength of the discontinuity will depend upon the nature of the surfaces in this area

of contact. The ratio of shear to normal load components is not affected by the uneven distribution of stress.

Amontons second law states that the frictional coefficient and hence friction angle for two surfaces in contact is independent of normal load. It has been shown, however, that rocks sometimes show a decreasing frictional coefficient with increasing normal stress, Maurer (1965), Murrell (1965), Hobbs (1970).

The uneven stress distribution along the base of a block on an inclined plane may, therefore, result in variable strength along the surface.

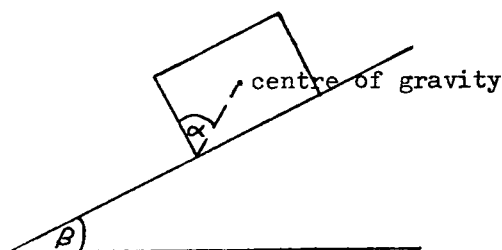
As a precursor to the tests using the vibrating test rig a series of experiments were carried out to investigate the reliability of the inclined plane sliding technique for determining angles of friction and the effect that uneven stress distribution due to variable block geometry and density may have on the results. The results obtained in this manner could then be used for comparison with angles of friction obtained from the vibrating test rig.

5.4 Inclined Plane Sliding tests using Wooden Sliders

Introduction

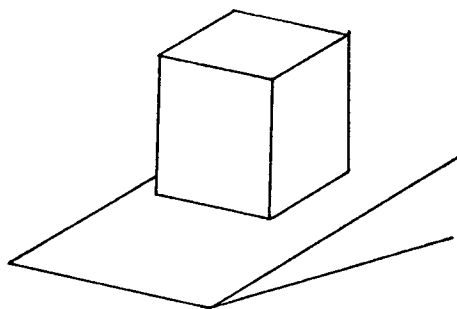
Bowden and Tabor (1964) report experiments carried out to investigate the friction of wood and concluded that friction arises from adhesion and deformation at the regions of real contact.

The experiments described below were performed to investigate repeatability of inclined plane sliding tests and the importance of stress distribution to the angle of sliding. They were not carried out with the purpose of determining a fundamental coefficient of friction for a specific wood and preparation of surfaces was therefore, not as exacting as reported by Bowden and Tabor. Each slider, however, was of the same wood and cut in a similar manner. All tests were carried out in air at room temperature. A set of orthogonal blocks of different heights but same basal dimensions were prepared. These blocks are described in terms of internal angle, α , i.e. the angle made between the leading edge normal to the shear plane and a line joining the centre of gravity and furthest projection on the plane in a down-slope direction, (see diagram below).

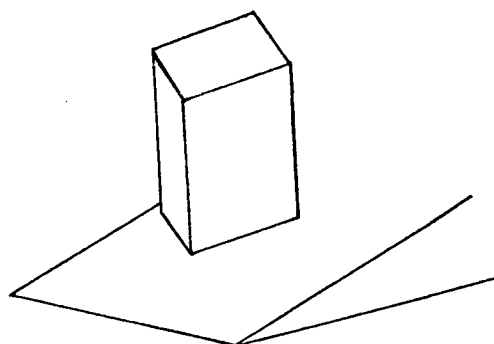


Tall homogeneous blocks, therefore, have low values of α , squat blocks have high values of α .

Blocks were placed on the plane in either position 1 or 2, edge or corner forward, respectively.



Position 1.



Position 2.

A first series of tests was carried out using each orthogonal block individually.

A second series of tests was carried out using columns of blocks in which the basal block was always the same. In this way the geometry of the sliding mass was varied whilst the sliding surface remained the same.

A third series used metal-weighted wood sliders, hence providing results for different normal loads to those in the first series of tests but for a similar range of geometries.

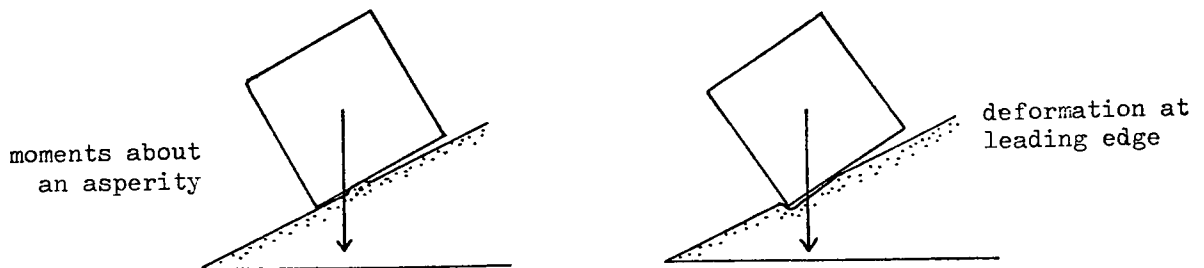
Experimental Procedure

Each block in turn was placed on a horizontal planar surface of the same wood, the inclination of which was gradually increased until sliding occurred. The angle at sliding was noted.

Experimental Observation

Several important features were noted during the experiments that are directly relevant to the interpretation of the results.

- i) Blocks were seen to tip forward, when the plane was still inclined at several degrees below the sliding angle. Light could occasionally be seen between the rear edges of the blocks and the plane. This was especially seen for taller blocks. Tipping forward may be caused by moments about asperities or by deformation at the leading edge of the block.



- ii) On sliding, blocks in position 1 normally slid the whole length of the plane. Blocks in position 2 generally slid at lower velocities and often stopped sliding after a short distance.
- iii) Throughout the series of tests it was found that if a block had failed at a certain angle, β , and this was then reduced by only a few degrees, to a position at which the block was previously stable, and the block then replaced, the block would no longer be stable. An attempt was made to stabilise blocks by pressing them onto the surface after replacing them at angles close to their angles of sliding but this did not seem to increase their resistance to sliding markedly.

Results

The results of sliding tests using wooden blocks and weighted wooden sliders are given in figures 5.2 to 5.9. Angle of sliding is plotted against the internal angle (α°) of each block in figures 5.2 to 5.6. Angle of sliding is plotted against the normal load component for each block at sliding, in figure 5.7. In figures 5.8 and 5.9, shear load components versus normal load components at sliding are plotted. Figure 5.8 contains data for light wooden blocks, figure 5.9 for the heavier metal weighted blocks.

Discussion

The scatter of data in figures 5.2 to 5.6 is considerable but three major trends are indicated:-

- i) The angle of sliding increases with increasing angle α (squatter blocks);
- ii) The angle of sliding for blocks placed in position 2 is lower than those in position 1. Figures 5.8 and 5.9 in which shear load components are plotted against normal load components for blocks in positions 1 and 2 emphasize this point:
- iii) The angle of sliding decreases with increasing normal load. This is illustrated by figures 5.6 and 5.9. The metal weighted blocks have a density approximately ten times that of the wooden blocks.

Figure 5.6 for position 1 or position 2 sliding is bilinear. If geometry were the only parameter affecting the angle of sliding all data for either position should lie on a single line. In fact blocks of the same geometry but different weight slide at different angles,

Angle of Sliding: Geometry - Wooden Blocks
Position 1.

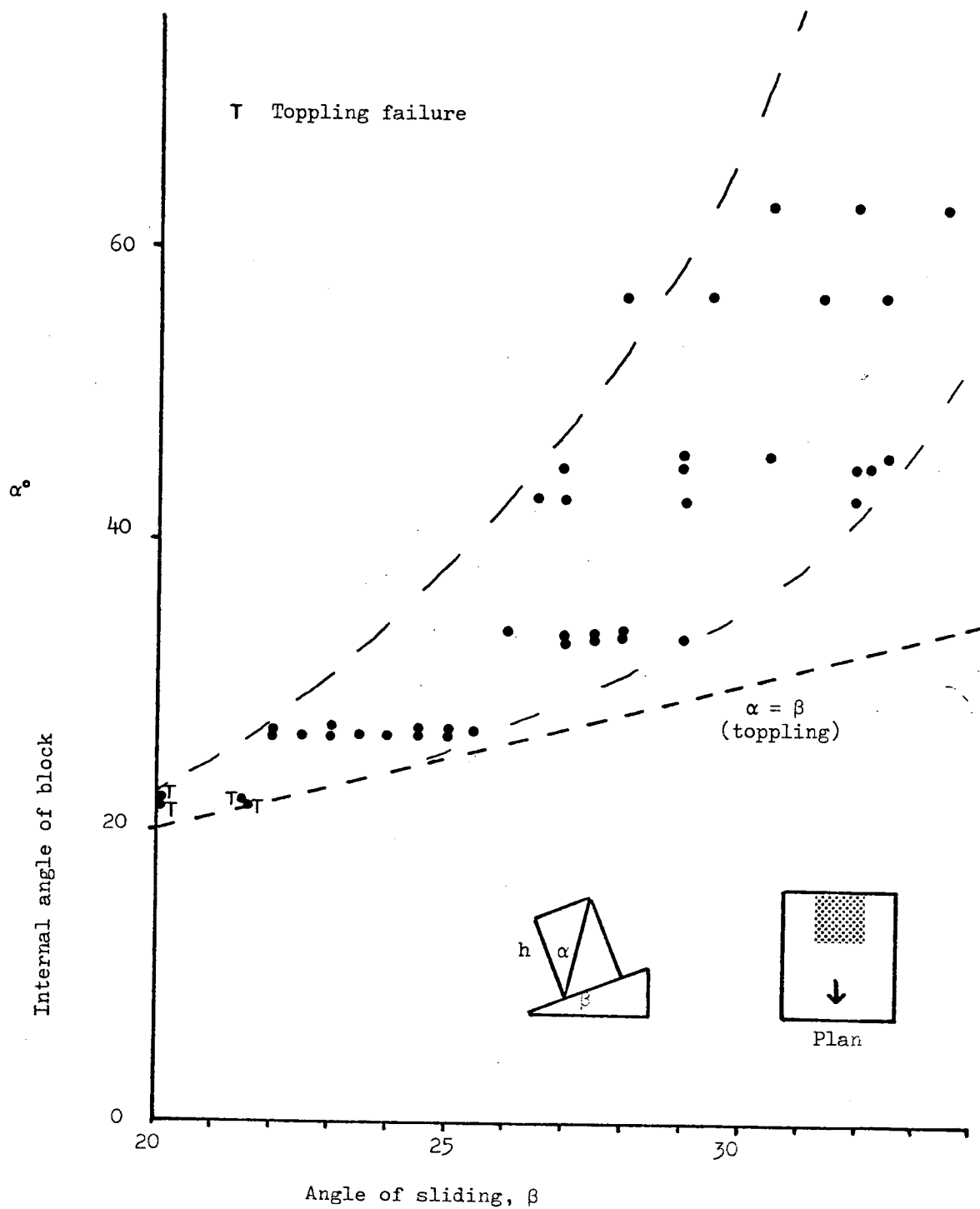


Figure 5.2

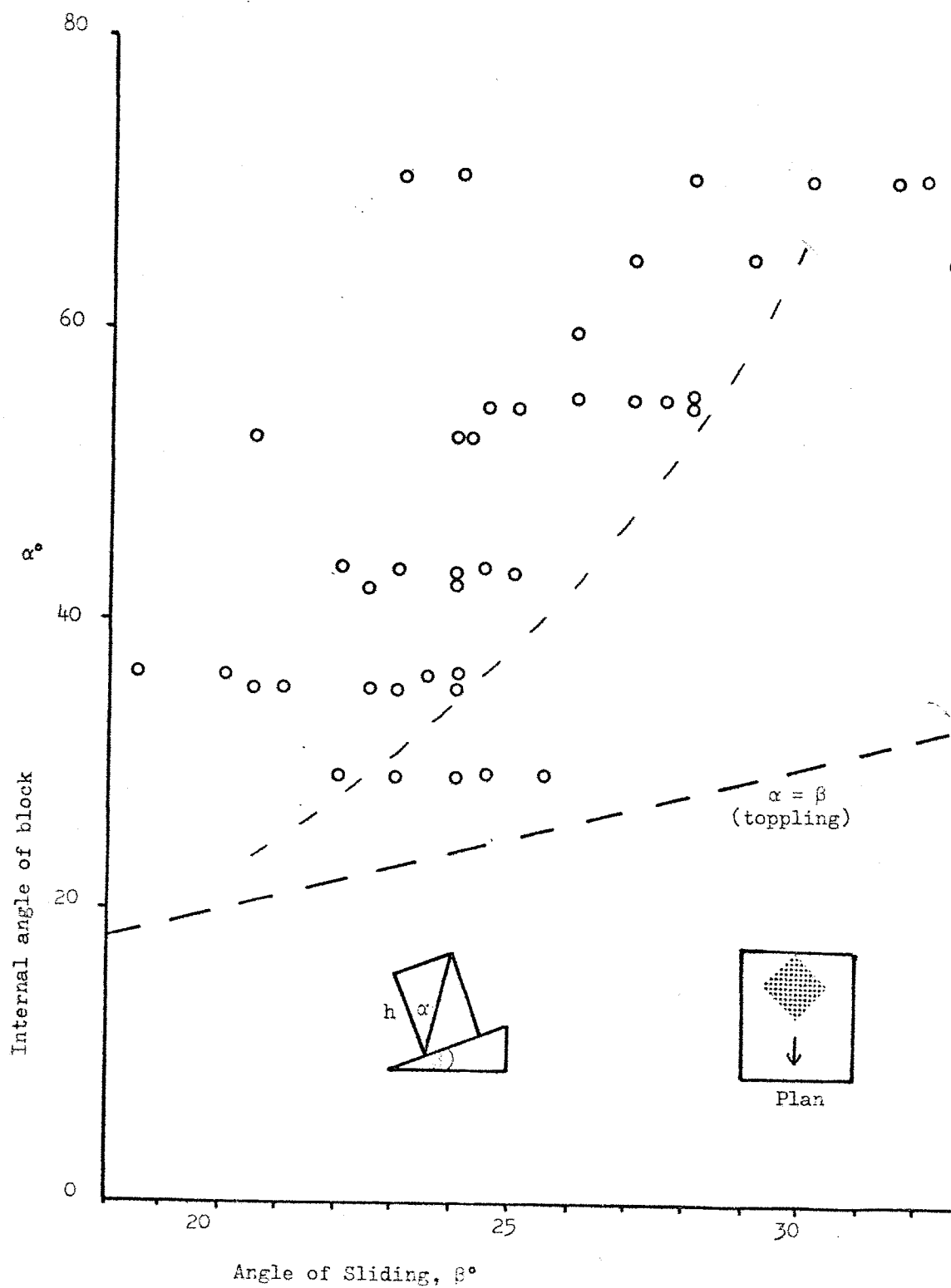


Figure 5.3 Angle of Sliding: Geometry - Wooden Blocks
Position 2.

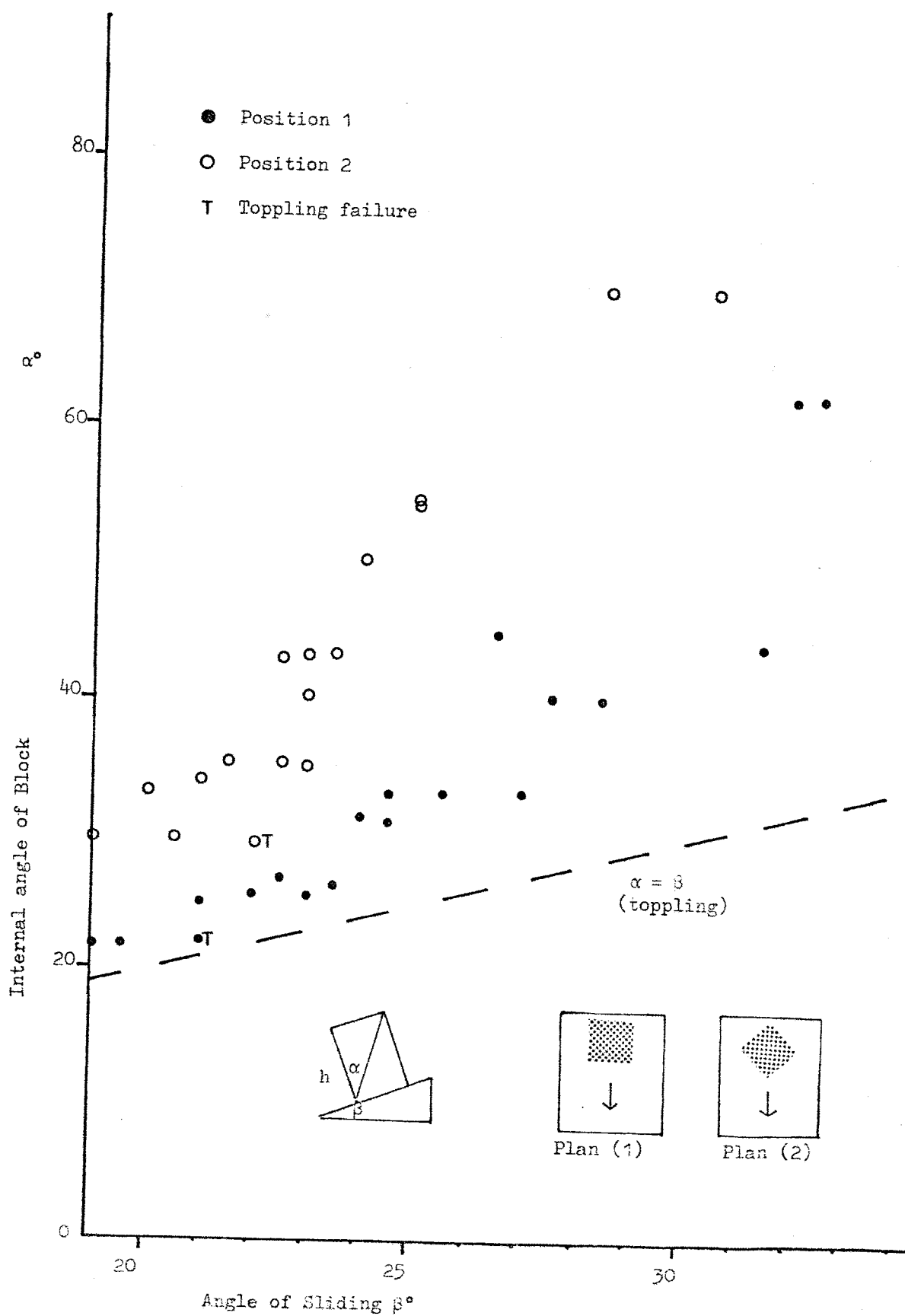


Figure 5.4 Angle of Sliding :Geometry - Wooden Blocks
Positions 1 and 2 (Sliding on common base surface)

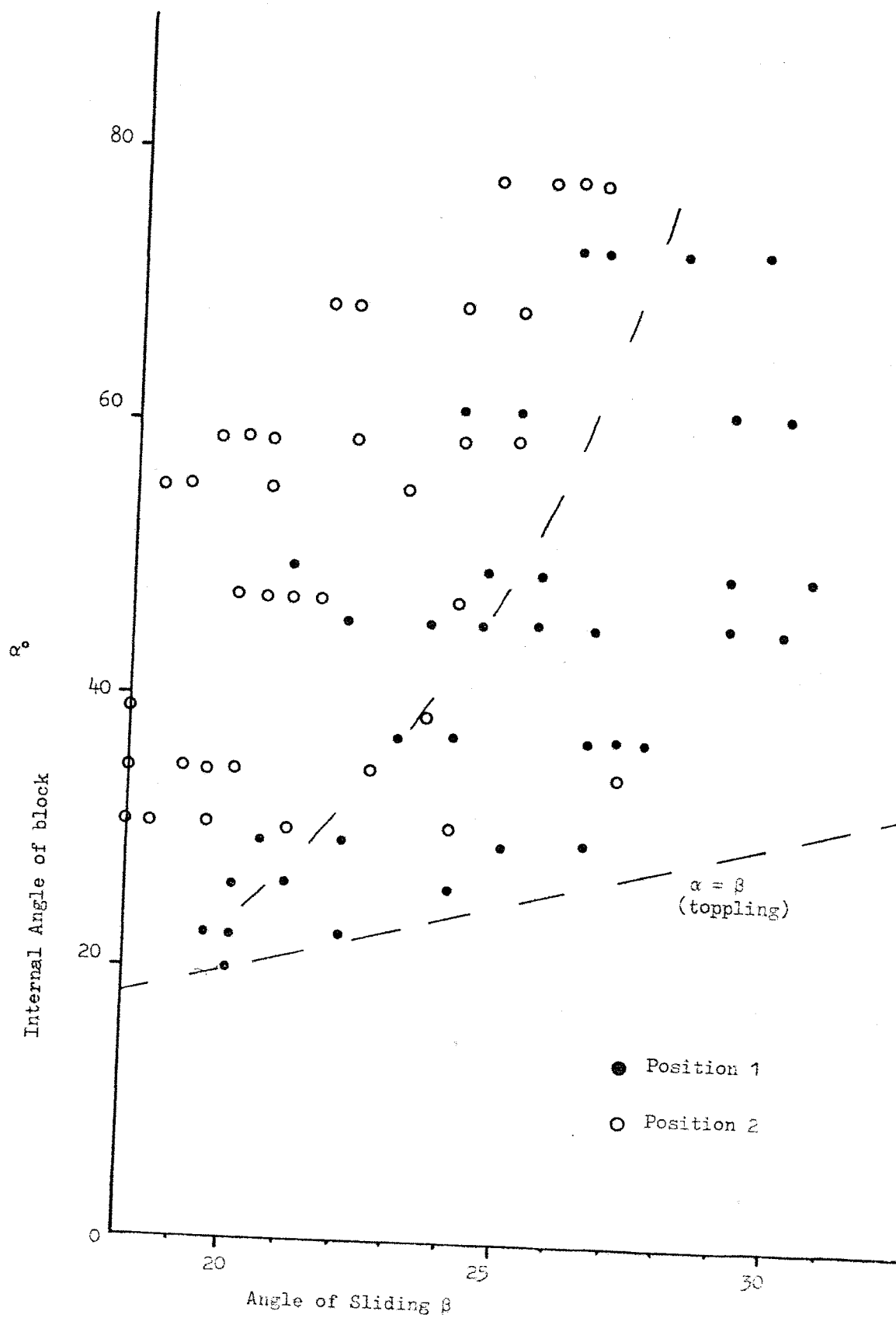


Figure 5.5 Angle of Sliding: Geometry - metal weighted wood sliders
Positions 1 and 2

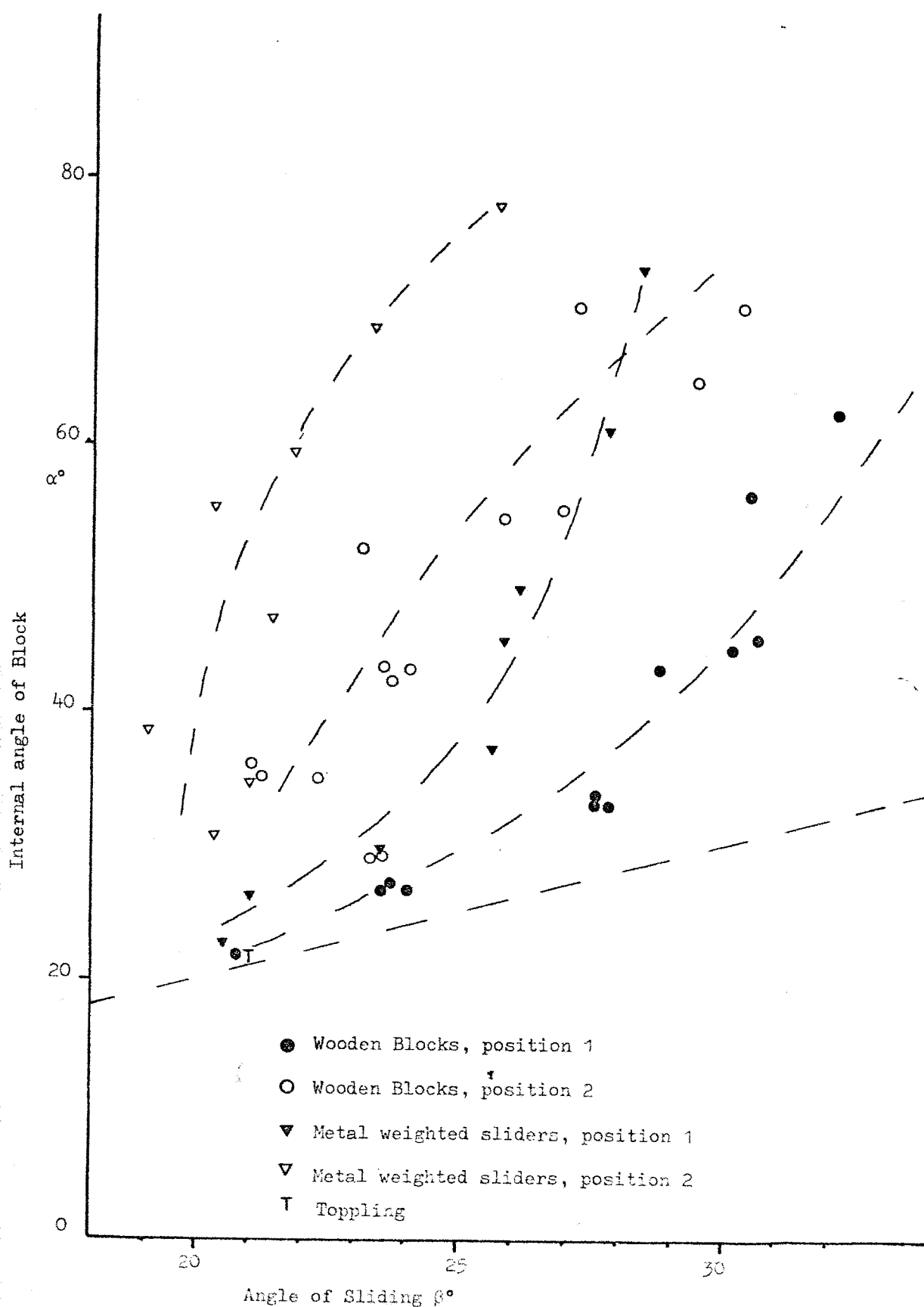


Figure 5.6 Average Values. Angle of Sliding: Geometry, Wooden Blocks and metal weighted wood sliders. Positions 1 and 2.

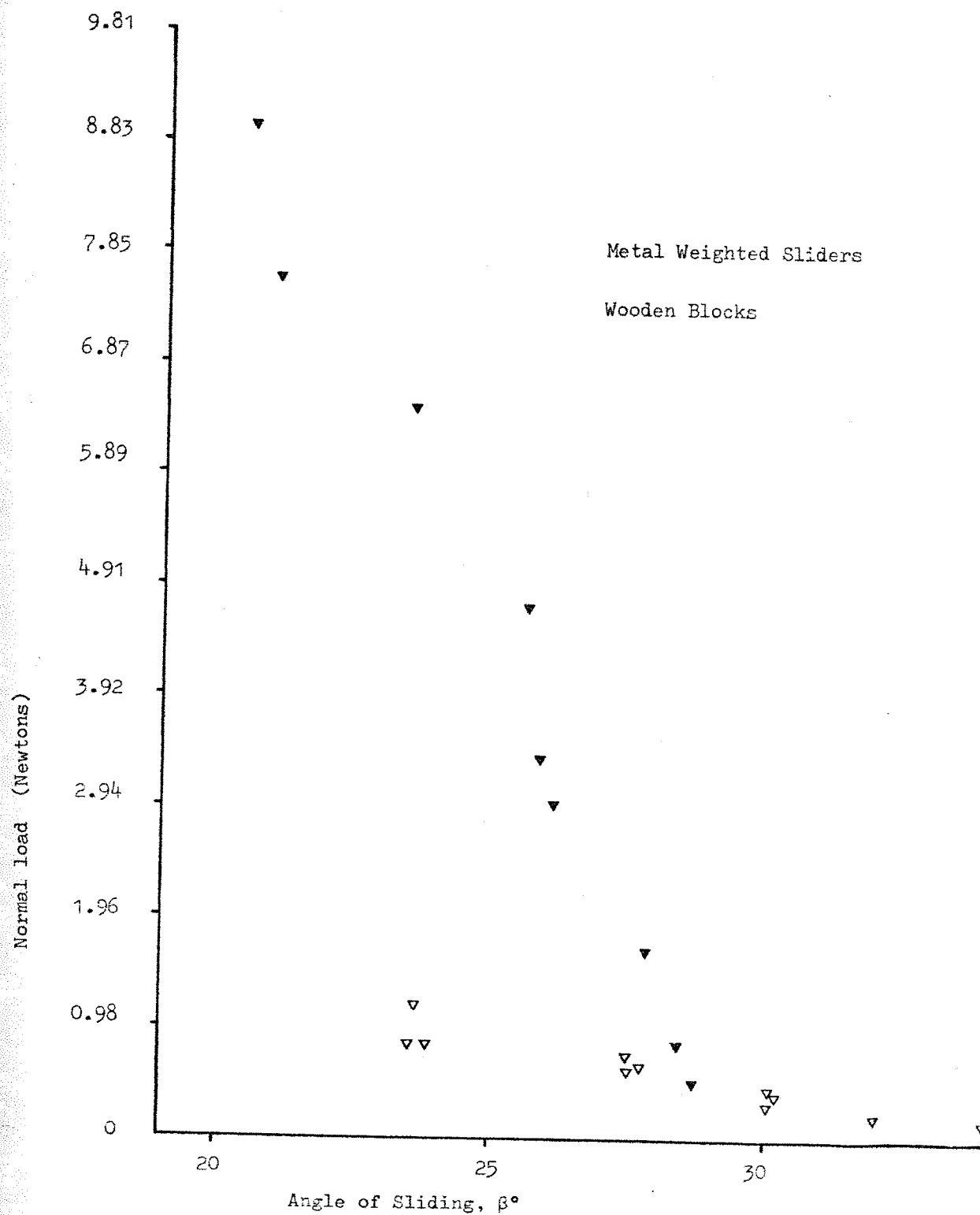


Figure 5.7 Angle of Sliding : Normal load at sliding.
Metal weighted sliders and wooden blocks, position 1.

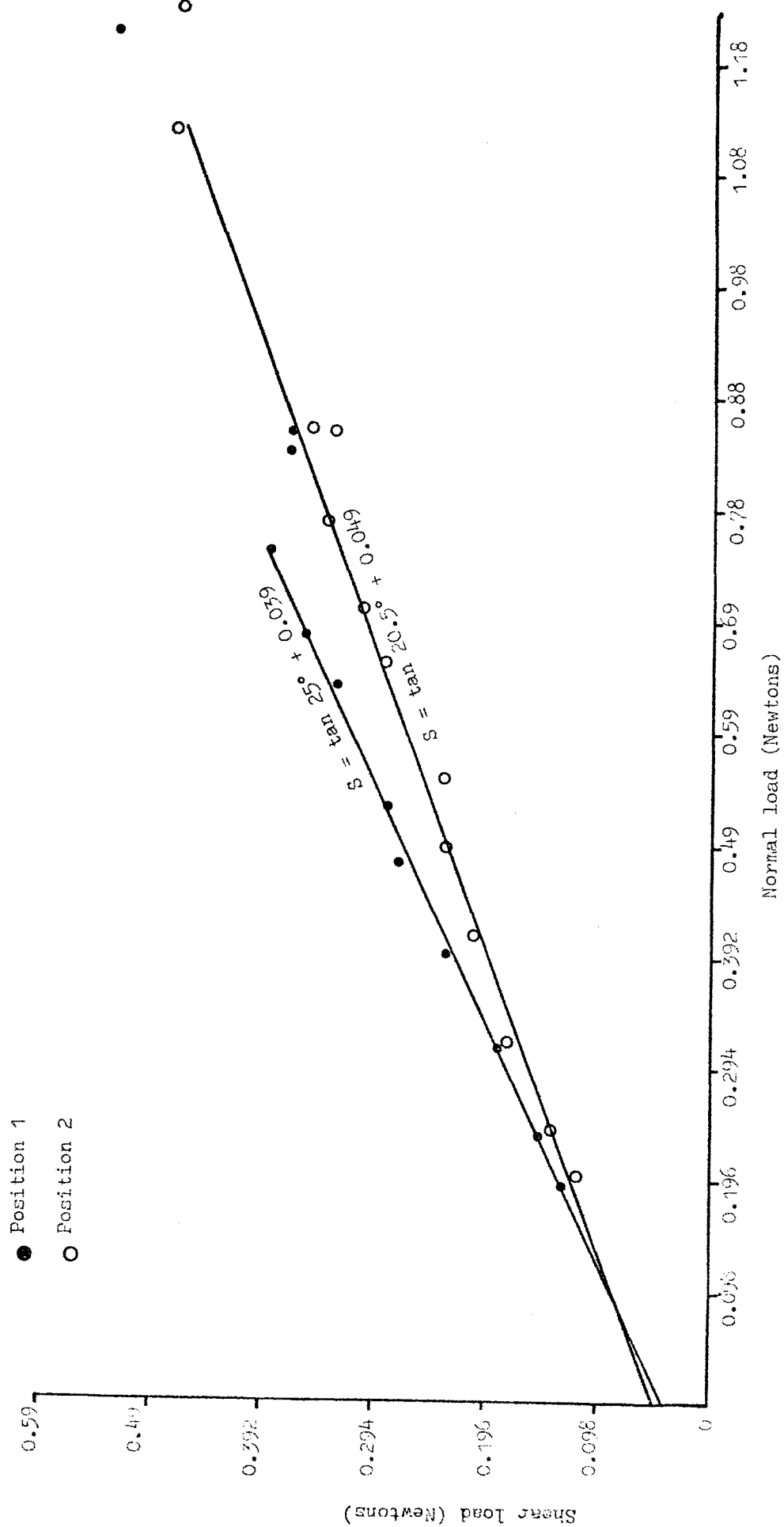


Figure 5.8 Shear load: Normal load components at sliding.
Wooden blocks positions 1 and 2.

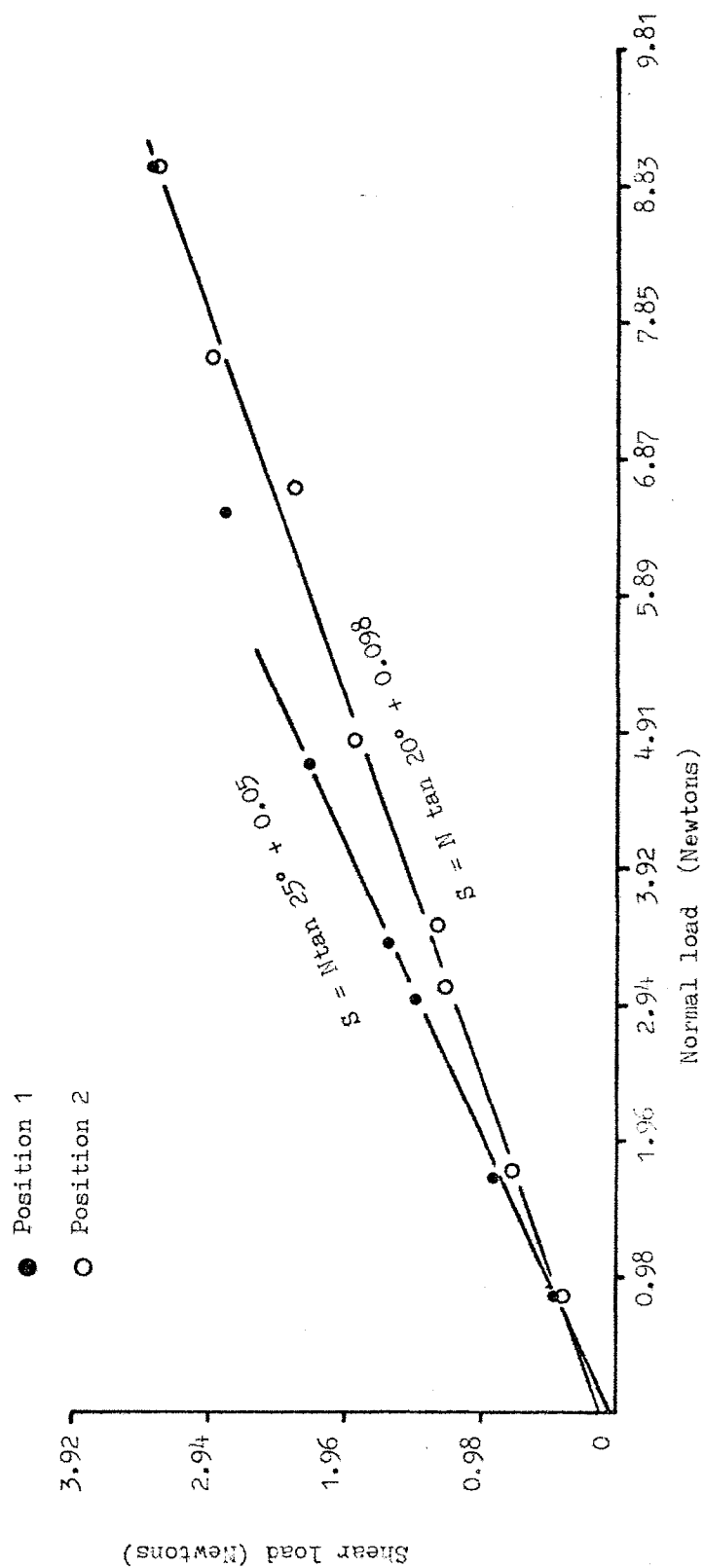


Figure 5.9 Shear load: Normal load components at sliding.
Metal weighted sliders positions 1 and 2

the heavier blocks at lower angles than the light blocks. In figure 5.7 the angles of sliding for metal weighted sliders and wooden blocks in position 1 are plotted against normal load at sliding. This graph is also bilinear illustrating that variation in normal load alone does not explain the results obtained.

It is difficult to formulate a theory that explains all the observed results and experimental observations presented above. A useful starting point, however, is to assume that the shear strength at sliding (S) can be expressed by an equation of the following type: (see chapter 4)

$$S = N \cdot \tan \phi + k \quad \dots \quad \dots \quad \dots \quad 5.1$$

Where N is the normal load component, ϕ an angle of friction for wood surfaces and k a 'cohesive' component.

Now considering blocks in position 1, there are two possible assumptions that may be made about the value of ϕ :-

- 1) ϕ must be equal to or less than 20.5° , the lower limit for sliding angle of blocks in position 1:
- 2) ϕ may be greater than 20.5° , (i.e. that certain blocks slid at angles of inclination less than the angle of friction for their surfaces).

Considering case 1, by taking the values for S and N for each block at sliding (table 5.1) and substituting these values into the above equation 5.1 for various possible values of ϕ ($17^\circ - 20.5^\circ$), values for k are obtained for each block at sliding, (table 5.2). Wooden blocks are numbered Wd(1 - 14), metal weighted wooden sliders, Wd/Mt(1 - 8).

The accompanying figures (figures 5.10, 5.11) show some interesting relationships. If k is dependant upon deformation then it should be related to the weight (W) of the block causing deformation. Similarly,

Block No.	β° av	S Newtons	N Newtons	W Newtons	α°
Wd 14	34.0	0.1289	0.1911	0.2306	62.92
Wd 13	32.0	0.1481	0.12371	0.2796	62.92
Wd 12	30.38	0.1836	0.3132	0.3630	56.45
Wd 11	30.63	0.2350	0.3968	0.4611	45.96
Wd 10	30.06	0.2703	0.4670	0.5396	45.00
Wd 9	28.63	0.2844	0.5210	0.5936	42.86
Wd 7	27.50	0.3285	0.6309	0.7113	33.90
Wd 6	27.75	0.3541	0.6729	0.7604	33.10
Wd 8	27.50	0.3896	0.7484	0.8437	33.42
Wd 3	23.88	0.3694	0.8343	0.9124	26.43
Wd 4	23.50	0.3697	0.8502	0.9271	26.57
Wd/Mt/8	28.40	0.4527	0.8372	0.9517	73.30
Wd 2	T 20.75	0.4293	1.1331	1.2117	21.65
Wd 5	23.63	0.5250	1.200	1.3098	27.11
Wd 1	T 20.75	0.5388	1.4221	1.5207	21.65
Wd/Mt/7	27.80	0.8831	1.6749	1.8935	61.34
Wd/Mt/6	26.10	1.4524	2.9648	3.3014	49.52
Wd/Mt/5	25.86	1.6377	3.3787	3.7547	45.77
Wd/Mt/4	25.60	2.2425	4.6805	5.19	37.43
Wd/Mt/3	23.50	2.8069	6.4555	7.0394	29.60
Wd/Mt/2	21.00	2.9183	7.6022	8.1931	26.26
Wd/Mt/1	20.50	3.3500	8.9600	9.5657	22.79

Table 5.1 Angle of sliding

β = Angle of sliding
 S = Shear load at sliding
 N = Normal load at sliding
 W = Weight of block
 α = Internal angle of block
 T indicates toppling failure

Block No. \ ϕ	17°	18°	19°	20°	21°	22°	25°
Wd 14	0.0705	0.0668	0.0631	0.0593	0.0555	0.0517	0.04
Wd 13	0.0756	0.0711	0.0665	0.0618	0.0571	0.0523	0.038
Wd 12	0.0878	0.0818	0.0758	0.0696	0.0634	0.0571	0.038
Wd 11	0.1137	0.1061	0.0984	0.0906	0.0827	0.0747	0.05
Wd 10	0.1275	0.1186	0.1095	0.1003	0.910	0.0816	0.053
Wd 9	0.1251	0.1151	0.1050	0.0948	0.0844	0.0739	0.041
Wd 7	0.1356	0.1233	0.1113	0.0989	0.0863	0.0736	0.034
Wd 6	0.1484	0.1355	0.1224	0.1092	0.0958	0.0822	0.04
Wd 8	0.1608	0.1464	0.1319	0.1172	0.1023	0.0872	0.04
Wd 3	0.1143	0.0983	0.0821	0.0657	0.0491	0.0323	-0.02
Wd 4	0.1098	0.0935	0.0770	0.0603	0.0433	0.0262	-0.02
Wd/Mt/8	0.1967	0.1807	0.1644	0.1480	0.1313	0.1144	0.062
Wd 2	0.0829	0.0611	0.0391	0.0169	-0.0057T	-0.0285T	-0.099
Wd 5	0.1581	0.1351	0.1118	0.0882	0.0644	0.0402	-0.03
Wd 1	0.1040	0.0767	0.0491	0.0212	-0.0071T	-0.0358T	-0.12
Wd/Mt/7	0.3710	0.3389	0.3064	0.2735	0.2402	0.2064	0.102
Wd/Mt/6	0.5460	0.4891	0.4315	0.3733	0.3143	0.2545	0.07
Wd/Mt/5	0.6047	0.5399	0.4743	0.4080	0.3407	0.2726	0.062
Wd/Mt/4	0.8115	0.7217	0.6309	0.5389	0.4458	0.3515	0.06
Wd/Mt/3	0.8333	0.7094	0.5841	0.4573	0.3289	0.1987	-0.203
Wd/Mt/2	0.5941	0.4482	0.3007	0.1513	0.0001	-0.1532	-0.62
Wd/Mt/1	0.6107	0.4387	0.2648	0.0888	-0.0894	-0.2701	-0.828

Table 5.2 Calculated values of k for various assumed values of ϕ°

T indicates toppling failure.

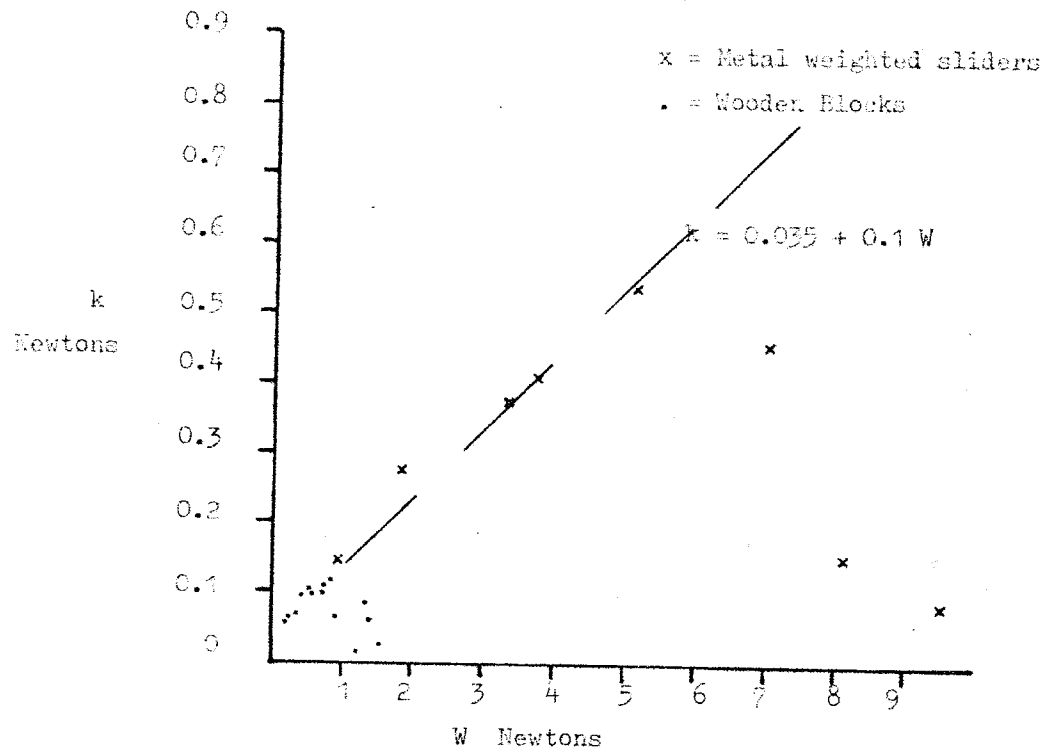
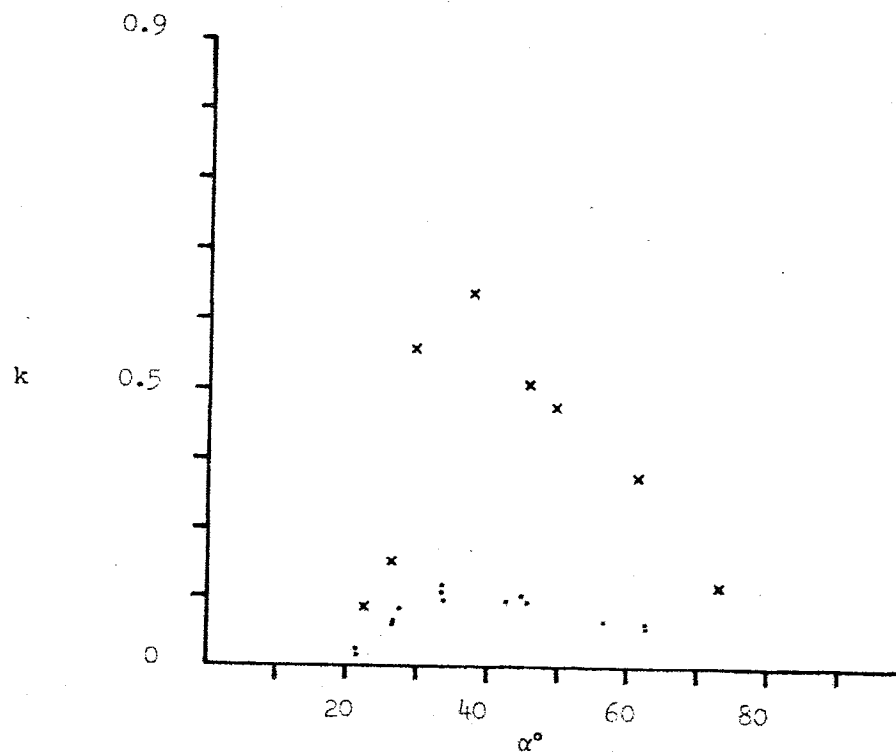


Figure 5.10 $k : W$ and $k : \alpha$ assuming $\phi = 20^\circ$



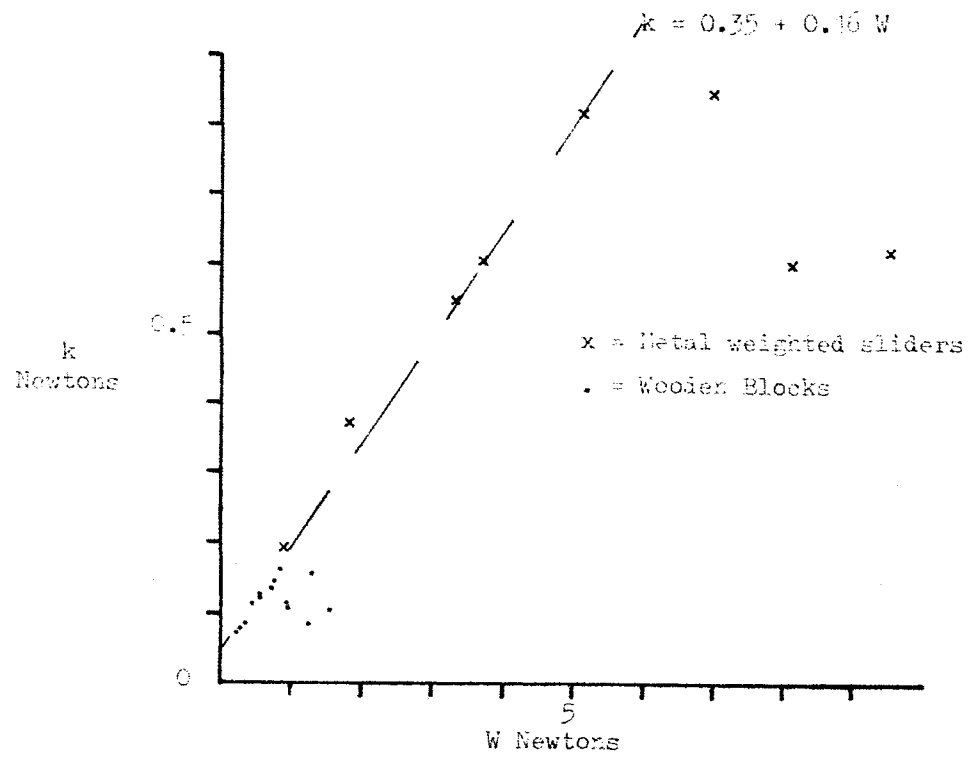
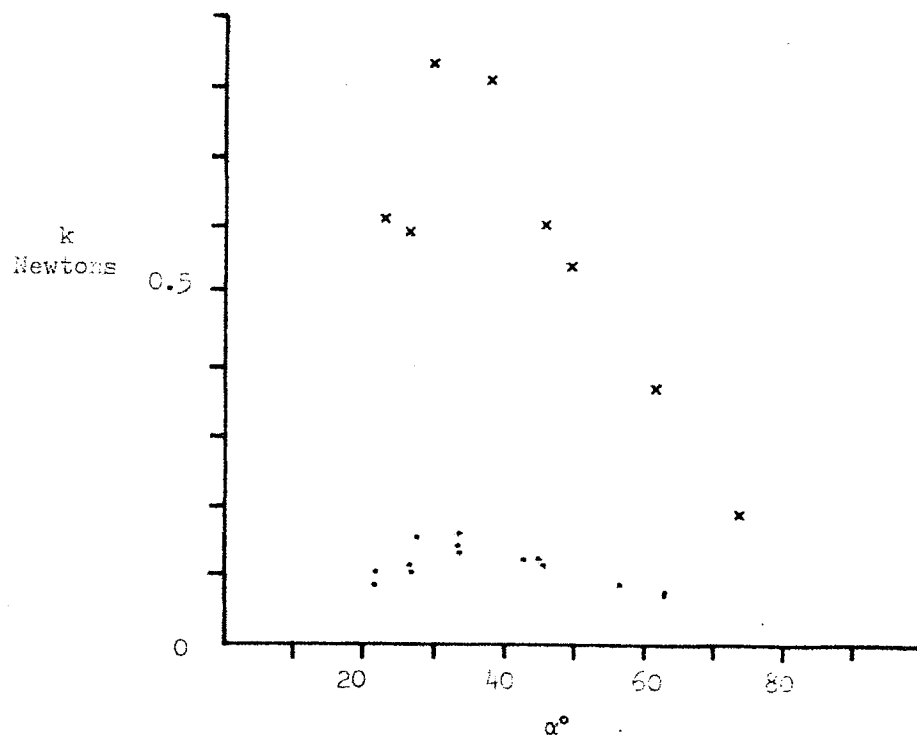


Figure 5.11 $k:W$ and $k:\alpha$ assuming $\phi = 17^\circ$



the uneven stress distribution (Appendix 1) might be expected to effect values for k especially where the angle of inclination is close to the internal angle of the block. Values of k for $\phi = 17^\circ$ and $\phi = 20^\circ$ have, therefore, been plotted in figures 5.10 and 5.11 against weight of block and internal angle separately.

It can be seen from figures 5.10 and 5.11 that for values of ϕ less than 21° , k has a linear relationship with W , where the internal angle of the block is greater than ϕ by approximately 13° . For blocks with internal angles closer to ϕ , k drops off in value rapidly with decreasing α .

The fact that k increases with W suggests that the dominant cause of k is deformation resulting in the block becoming embedded into the surface of the basal plane. Presumably with blocks of internal angles close to ϕ , the uneven distribution of stress, (see Appendix 1, figure A.1.2), results in an unstable situation and k is unable to develop its full value.

In summary if this hypothesis is correct then for the blocks tested, angles of sliding depend upon the weights of the blocks and internal angles of the blocks:

$$\text{then } S = N \cdot \tan \phi + k$$

Remembering that the values for ϕ and k derived from these experiments relate to wood, then it can be seen that ϕ can take any value from $0 - 20.5^\circ$. For $\phi = 17^\circ$, $k = 0.35 + 0.16W$ Newtons

$$\text{where } \alpha \gg \phi + 13^\circ$$

for $\phi = 20^\circ$

$$k = 0.35 + 0.10W \text{ Newtons}$$

$$\text{where } \alpha \gg \phi + 13^\circ$$

Considering the situation for blocks with angles α close to ϕ it may be seen, especially in the lower part of fig. 5.10 that as α approaches ϕ , k drops to zero. It might be expected, therefore, that blocks sliding on their edges, representing the case of all blocks where $\alpha = \beta$, may slide where $\beta = \phi$.

To investigate this, edge sliding tests were carried out. The results of twelve edge sliding tests gave an average angle of sliding of 21° with a standard deviation of 1.47° . Table 5.2 shows that for $\phi = 21^\circ$, three blocks have negative values of k . In fact two of these blocks, Wd2 and Wd1 toppled rather than slid. The other block Wd/Mt 1 slid at 20.5° . Assuming that the behaviour of this block may also be represented by equation 5.1, it may be concluded that, making the assumption that $\phi \leq$ the lowest value of β observed, ϕ has an approximate value of between 20° and 21° with k taking the values given in table 5.2.

Considering case 2, where it is no longer assumed that ϕ must be smaller than β then values of ϕ and k , where k is constant, may be obtained directly from figures 5.8 and 5.9. It is found that data for blocks in position 1 with internal angles greater than 30° lie on straight lines such that

$$S = N \tan 25^\circ + 0.039 \quad (\text{fig. 5.8})$$

$$\text{and } S = N \tan 25^\circ + 0.05 \quad (\text{fig. 5.9})$$

Similarly if k is obtained for each block numerically where $\phi = 25^\circ$ (see table 5.2) it is found that k is fairly constant for all blocks with a greater than 30° , with a mean of 0.052 and standard deviation of 0.018.

Confirmation of this result was obtained by direct shear tests (figure 5.12) which gave a value of $\phi = 26^\circ$ and $k = 0.05$ for position 1 sliding. It is clear that blocks with internal angles less or slightly

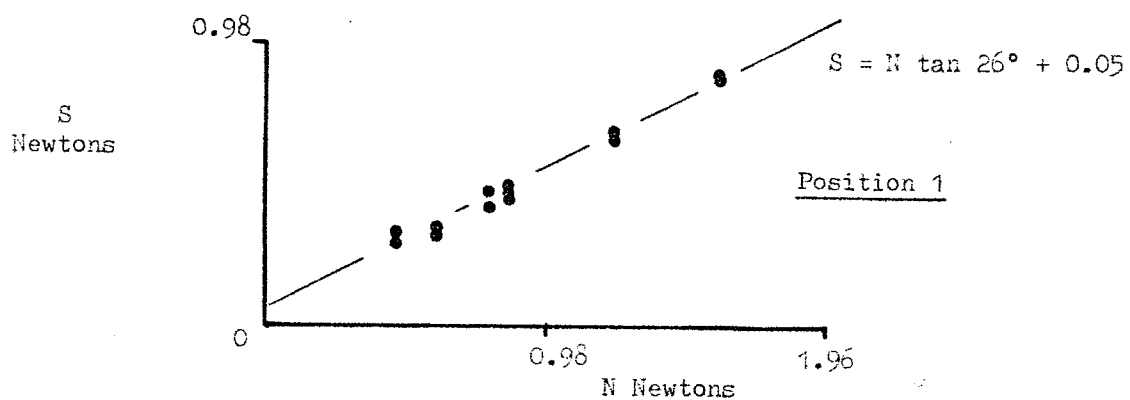


Figure 5.12 S:N at Sliding for Wooden surfaces in direct shear
(Position 1)

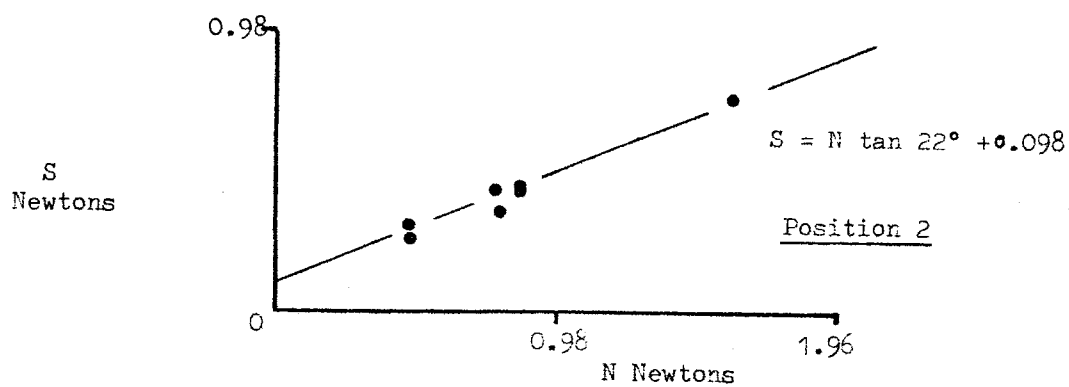


Figure 5.13 S:N at Sliding for wooden surfaces in direct shear.
(Position 2)

greater than ϕ slid at angles (β) less than ϕ . Presumably as angle β approaches α (towards a toppling mode of failure) then the blocks become unstable and lose both cohesion and a proportion of their resistance due to friction.

In summary the data obtained by inclined plane sliding for blocks in position 1 may be explained by two hypotheses.

Hypothesis 1. That ϕ , the angle of friction, is lower than the lowest angle of sliding observed. If this is the case then it is shown that a cohesive term (k) must be introduced to explain the results, and that the cohesive term comprises a constant plus a variable which is directly proportional to the normal load applied. For tall blocks with internal angle close to ϕ , the cohesive term (k) no longer increases with normal load, but drops off rapidly, being zero when $\alpha = \phi$.

Hypothesis 2. That ϕ may be greater than the angle of sliding of some blocks. In this case k may be taken as a constant for blocks with $\alpha > \phi + 5^\circ$. Blocks sliding at lower angles than ϕ evidently lose cohesion and a proportion of frictional resistance.

Evidence for deciding which of these hypotheses is most likely comes from the data for sliding in position 2.

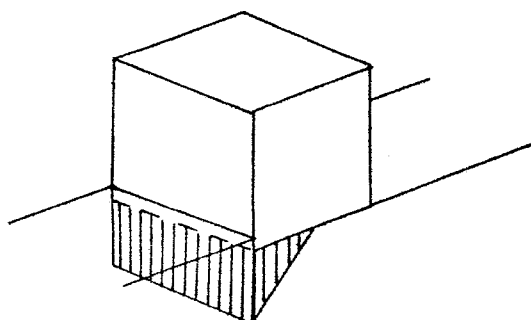
If it is again assumed that the shear strength at sliding may be expressed by equation 5.1, then it is logical to also assume that the value of ϕ for positions 1 and 2 will be the same, the blocks used for sliding being the same in each case. Variations in sliding angle for blocks in both positions may therefore be due to different 'cohesive' mechanisms in each case.

Figures 5.8 and 5.9 give shear load versus normal load data at sliding for each block in position 2. It is clear from these figures that ϕ cannot equal 25° for position 2 sliding unless one introduces a negative cohesive component, becoming increasingly negative with increasing normal load. Figure 5.13 containing data for direct shear tests in position 2 similarly shows a lower angle for ϕ .

Therefore, if ϕ is a constant for both position 1 and position 2 sliding, it is unlikely that hypothesis 2, above, is true.

Reconsidering position 2 data in terms of hypothesis 1, the values of ϕ for position 2 sliding in figures 5.8 and 5.9 agree well with that deduced for position 1 sliding. It is also apparent that there is no reduction in k for blocks in position 2 where α approaches ϕ . Both ϕ and k are constants. The higher angle of sliding seen for blocks with smaller α values, are simply a result of the cohesive constant resisting a greater proportion of the shear load for lighter blocks than for heavy blocks, and has no relation to α except that α is related to weight of the blocks.

The difference between angles of sliding for blocks in position 1 and position 2 is, therefore, due to the variable component of cohesion in position 1. In position 1 lines of equal stress run across the base of the block parallel to the front edge. It may be summarised, therefore that a linear barrier is set up due to deformation at the front edge of the block.



In position 2 the more streamlined shape with concentration of stress at the leading corner, rather than across any edge, prevents the formation of any such deformation barrier.

This possible cause of resistance explains why blocks in position 1 slide faster than in position 2.

In position 1 the peak shear strength is due to friction plus a cohesive constant plus a variable cohesive element due to deformation. Once the deformation factor has been overcome then the block will have a suddenly reduced shear strength; i.e. for these experiments,

$$S = N \cdot \tan \phi + 0.35 + C_w \rightarrow S = N \cdot \tan \phi + 0.35$$

where C_w is a constant.

Position 2 sliding without this deformation component will have the same strength at and during sliding, and hence slide more evenly. Any minor obstacle in the path of the sliding block may cause the block to stop sliding.

Conclusions

Experiments in inclined plane sliding using wooden surfaces, have indicated a relationship between the angle of sliding and weight and geometry of blocks. A lowering of the angle of sliding is also seen for blocks with corners pointed down slope relative to those sliding edge on.

The data obtained may be explained where the shear strength is represented by an equation such that

$$S = N \cdot \tan \phi + k$$

where k is a variable cohesive component.

It is found that all the data for the wooden surfaces tested agrees with a value of ϕ of between 20 and 21°. k is a constant for position 2

sliding and takes the value 0.098 Newtons for the wooden surfaces used.

For position 1 sliding k is variable and is the sum of a constant (approximately 0.05 N) plus a proportion of the weight of the block.

As the angle of the block increases towards ϕ however, the cohesive component begins to decrease, becoming zero where $\alpha = \phi$.

It is suggested that the variable component of k for position 1 sliding is due to deformation resulting in a ridge preventing sliding.